

# Massive charged scalar field in a Reissner-Nordstrom black hole background: quasinormal ringing

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## Abstract

We compute characteristic (quasinormal) frequencies corresponding to decay of a massive charged scalar field in a Reissner-Nordstrom black hole background.

It proves that, contrary to the behavior at very late times, at the stage of quasinormal ringing the neutral perturbations will damp slower than the charged ones. In the limit of the extremal black hole the damping rate of charged and neutral perturbations coincides. Possible connection of this with the critical collapse in a massive scalar electrodynamics is discussed.

Black hole (BH) quasinormal modes (QNM's) govern the decay of perturbations at intermediate times and are important when studying dynamics of black holes and external fields around them. Now, the interest in quasinormal modes is stipulated by three their features. The first is connected with possibility of observing of QNM's, and of obtaining the "footprint" of a black hole, with the help of a new generation of gravitational antennae which are under construction (see [1] for a review). The second concerns the Anti-de-Sitter/Conformal Field Theory (AdS/CFT) correspondence [2], from which it follows that a large black hole in AdS space corresponds to an approximately thermal states in the CFT. Thus the decay of the BH perturbations can be associated with the return to thermal equilibrium of the perturbed state in CFT [3]. Note that in AdS space, quasinormal modes of black holes govern the decay of a field at late times as well, since there is no power-law tails and the decay is always exponential (see [3] and references therein). The third feature is related to possible connection of BH QNM's in some space-time geometries with the Choptuik scaling [3], [4], [5], [6].

When Horowitz and Hubeny calculated massless scalar quasinormal modes for a four-dimensional SAdS black hole they found that for intermediate black holes, i.e. when the horizon radius is of order the Anti-de Sitter radius (see also [7]), the striking approximate relation takes place

$$\omega_{Im} \sim \frac{1}{\gamma} r_+, \quad (1)$$

where  $r_+$  is the horizon radius,  $\gamma$  is the Choptuik scaling parameter for a black hole being formed from a collapse of a scalar field in free space. Then, exact connection between the Choptuik scaling parameter and the imaginary part of the QNF's was obtained for the

massive scalar field in the background of the three-dimensional BTZ BH [4]. It gives us a hope that such a connection between characteristic frequencies of black holes, at least in special space-time geometries, and parameters describing situation near the threshold formation of a black hole exists for all black holes. Thus knowledge of such relations would be a most useful, since calculation of quasinormal modes is much simpler work than numerical simulation of gravitational collapse.

Then it was observed in [5] that since the nearly extremal RN BH background is effectively described by  $AdS_2$  black holes under a spherically symmetrical dimensional reduction [8], the relation between the scalar QN modes of the nearly extremal RN BH and the Choptuik scaling may be obtained.

Recently it has been found that at the stage of quasinormal ringing the neutral perturbations, corresponding to a massless scalar field, will damp slower than the charged ones for RN, RNAdS, and dilaton black holes [6]. At very late times, on contrary, the charged perturbations will damp slower, thereby dominating in a signal as was shown in [9]. In addition the damping rates (imaginary part of  $\omega$ ) of charged and neutral quasinormal modes for massless scalar field coincide for the nearly extreme RN black hole [6]. This fact, although not understood, may be connected with the Choptuik scaling if remembering that the Choptuik scaling parameter  $\gamma$  is the same (0.37) both for charged [10] and neutral [11] massless scalar field collapse. The case of a massive scalar field is more complicated from a critical collapse view. There both types of behavior appear (see [12] and references therein), I (with a mass gap) and II (when the black hole being formed may be of infinitesimal mass) depending on ratio of the length scale of the initial data to the inverse Compton wavelength characteristic of the field. Yet the critical collapse was studied only for massless electrodynamics and for a massive scalar field. The complete picture of critical collapse for massive scalar electrodynamics is lacking as far as we are aware. This gives us one of the reasons for studying the quasinormal spectrum associated with decay of a charged massive scalar field.

The other reason is to find out how a massive scalar field, interacting also electromagnetically with a charged black hole, decay between an initial burst of radiation and the late time tails. In this direction the evolution of a neutral massive scalar field at late times was studied in [13] for a Schwarzschild background, in [17], [18], [19] for a RN background, and in [14] for the dilaton black hole. At the stage of quasinormal ringing, decay of neutral massive scalar field was studied in [20] for Schwarzschild and Kerr black holes. The quasinormal modes of dilaton black holes were studied in [6], [15], [16].

The complete picture of a charged massive scalar field evolution in a charged black hole background is lacking at either stages.

We shall consider the evolution of the massive charged scalar field in the background of the Reissner-Nordstrom metric:

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2d\Omega_2^2, \quad (2)$$

where  $f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$ . The wave equation of the complex scalar field has the form:

$$\phi_{;ab}g^{ab} - ieA_ag^{ab}(2\phi_{;b} - ieA_b\phi) - ieA_{a;b}g^{ab}\phi + \mu^2\phi = 0, \quad (3)$$

here the electromagnetic potential  $A_t = C - \frac{Q}{r}$ ,  $C$  is a constant. After representation of the charged scalar field into spherical harmonics the equation of motion takes the form:

$$\psi_{,tt} + 2ie\frac{Q}{r}\psi_{,t} - \psi_{,r^*r^*} + V\psi = 0, \quad (4)$$

where

$$V = f(r) \left( \frac{l(l+1)}{r^2} + \frac{2M}{r^3} - \frac{2Q^2}{r^4} + \mu^2 \right) - e^2 \frac{Q^2}{r^2}, \quad (5)$$

the tortoise coordinate is defined by the relation  $dr^* = \frac{dr}{f(r)}$ , and  $\psi = \psi(r)e^{-i\omega t}$ ,  $\omega = \omega_{Re} - i\omega_{Im}$ . The effective potential goes to a constant at the horizon and at infinity. We compute the quasinormal frequencies stipulated by the above potential using the third order WKB formula of S.Iyer and C.Will [21]:

$$\frac{iQ_0}{\sqrt{2Q_0''}} - \Lambda(n) - \Omega(n) = n + \frac{1}{2}, \quad (6)$$

where  $\Lambda(n)$ ,  $\Omega(n)$  are second and third order WKB correction terms depending on the potential  $Q$  and its derivatives in the maximum. Here  $Q = -V + \omega^2 - 2\frac{eQ}{r}\omega$ . Since  $Q$  generally depends on  $\omega$ , the procedure of finding of the QN frequencies is the following: one fixes all the parameters of the QN frequency, namely, the multipole index  $l$ , the overtone number  $n$ , of the black hole  $Q$  and  $M$ , and of the field  $m$  and  $e$ ; then one finds the value of  $r = r_0$  at which  $Q$  attains a maximum as a numerical function of  $\omega$  and substituting it into the formula (6) one finds  $\omega$  which satisfies the equation (6). Note that the effective potential, being frequency dependent, is, generally, complex. Nevertheless, following [21], we treat  $\omega$  as if it were real when finding  $r_0$  and only continue Eq.(6) into the complex frequency plane. We deal here with  $n = 0$  modes as those dominating in a signal.

First we examine the behaviour of the neutral massive scalar field in a RN background. Recently in [18] through numerical simulation of the wave equation solution it has been obtained that the relaxation process depends on the value  $M\mu$ : when  $M\mu \ll 1$  the relaxation depends on the field parameters and does not depend on the spacetime parameters, while at  $M\mu \gg 1$  the dependence on the black hole parameters appears. At the stage of quasinormal oscillations the evolution of the massive scalar field governed by QNF's depends both on black hole and field parameters.

QNF's for different values of  $\mu$  are presented on Fig.1- 3 for  $l = 1, 2, 3$ . The  $l = 0$  modes are not given here, since the WKB accuracy is not sufficient in this case (nevertheless the main conclusions as to the quasinormal behaviour conserve in this case as well). The real part of the quasinormal frequency, i.e. the oscillation frequency, grows with increasing of the mass field  $\mu$ , while the imaginary part of  $\omega$ , representing the damping rate, falls down. Note that we are limited to the low-lying quasinormal frequencies whose real parts correspond to tunneling near the potential barrier ( $\omega^2 \approx V_{max}$ ). In general, the maximum value of  $\mu$  (and of  $e$  for charged scalar field) depends upon the mode under consideration [20].

Next, we investigate the massive charged scalar field case. The real part of  $\omega$  as a function of imaginary part of  $\omega$  is plotted for different  $l$  and  $Q$  for  $\mu = 0.1$  on Fig.4-6. We see that the neutral perturbations ( $e = 0$ ) have less imaginary part than that of the charged perturbations. Thus the charged perturbations decay faster. This repeats the behaviour of the massless case [6]. When increasing charge  $Q$ ,  $\omega_{Im}$  grows up to some maximum near  $Q = 0.8M$  and then falls down, while  $\omega_{Re}$  increases monotonically. The more  $e$  the less explicit this maximum.

In addition for the near extremal black hole the imaginary parts of charged and neutral perturbations tend to coincide as in a massless case. In other words for the near extremal

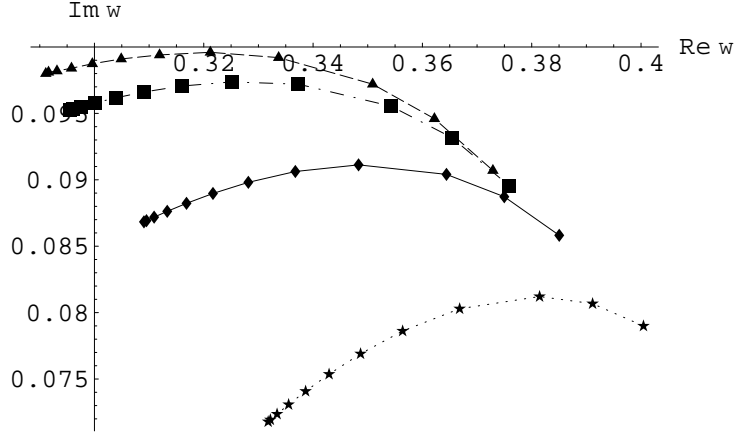


Figure 1: RN QNF's for  $l = 1$ ,  $n = 0$ ,  $\mu = 0, 0.1, 0.2, 0.3$ ,  $Q$  runs the values  $0, .1, .2, .3, .4, .5, .6, .7, .8, .9, .95, .99$

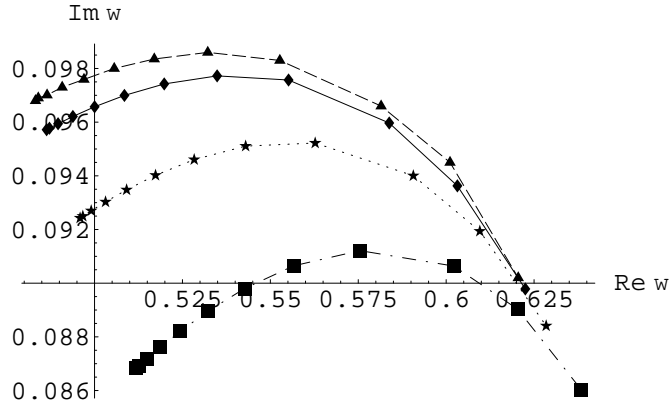


Figure 2: RN QNF's for  $l = 2$ ,  $n = 0$ ,  $\mu = 0, 0.1, 0.2, 0.3$ ,  $Q$  runs the values  $0, .1, .2, .3, .4, .5, .6, .7, .8, .9, .95, .99$

black hole  $\omega_{Im}$  does not depend on  $e$  for a charged massive scalar field. In this connection if one supposes some kind of relation between  $\omega_{Im}$  of the nearly extremal RN BH and the parameter of the critical collapse in a massive scalar electrodynamics, then he could expect that  $\gamma_{massive}$  being dependent on mass field  $\mu$  does not depend on  $e$  in the region where type II [12] critical behaviour occurs. Yet, at present we have much more questions than answers here.

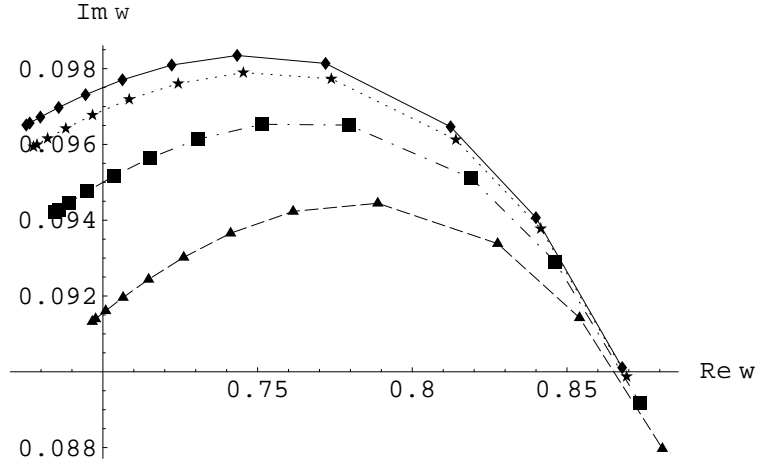


Figure 3: RN QNF's for  $l = 3$ ,  $n = 0$ ,  $\mu = 0, 0.1, 0.2, 0.3$ ,  $Q$  runs the values  $0, .1, .2, .3, .4, .5, .6, .7, .8, .9, .95, .99$

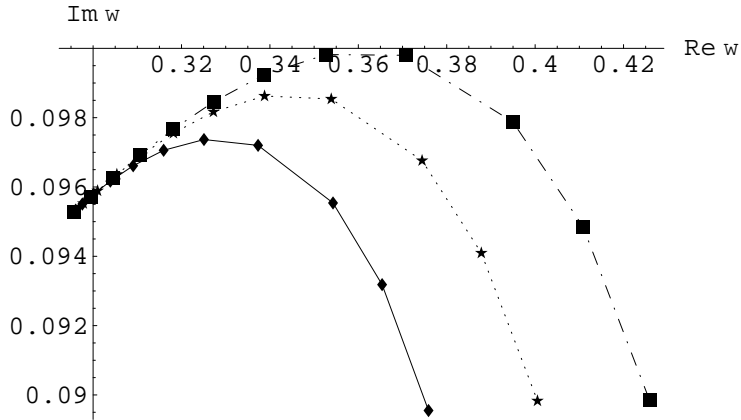


Figure 4: RN QNF's for  $l = 1$ ,  $n = 0$ ,  $e = 0$  (bottom),  $0.05$ , and  $e = 0.1$  (top),  $Q$  runs the values  $0, .1, .2, .3, .4, .5, .6, .7, .8, .9, .95, .99$ ,  $\mu = 0.1$

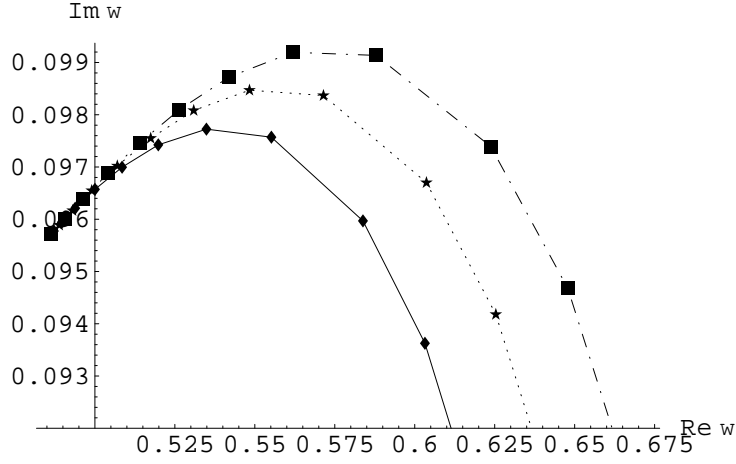


Figure 5: RN QNF's for  $l = 2$ ,  $n = 0$ ,  $e = 0$  (bottom),  $0.05$ , and  $e = 0.1$  (top),  $Q$  runs the values  $0, .1, .2, .3, .4, .5, .6, .7, .8, .9, .95, .99, \mu = 0.1$

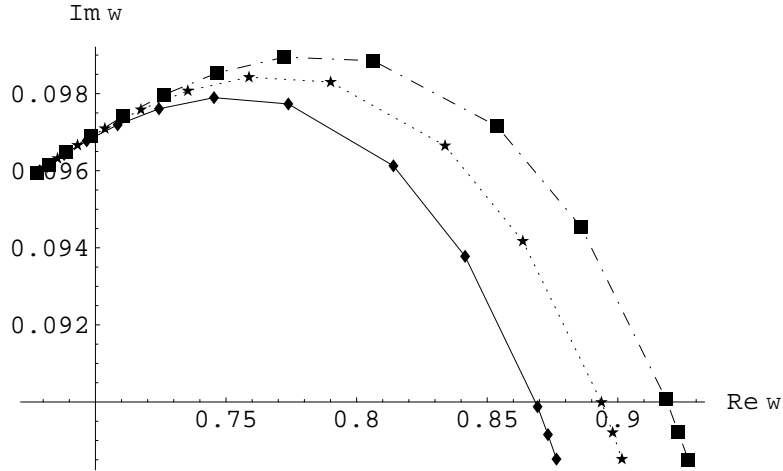


Figure 6: RN QNF's for  $l = 3$ ,  $n = 0$ ,  $e = 0$  (bottom),  $0.05$ , and  $e = 0.1$  (top),  $Q$  runs the values  $0, .1, .2, .3, .4, .5, .6, .7, .8, .9, .95, 0.995, 0.999, \mu = 0.1$ . At  $Q = 0.999$   $\omega_{Im} = 0.088517$  at  $e = 0$ ,  $\omega_{Im} = 0.088512$  at  $e = 0.5$ , and  $\omega_{Im} = 0.088470$  at  $e = 0.1$ . Thus the coincidence of  $\omega_{Im}$  for different  $e$  in the near extremal regime is well within a third order WKB accuracy.

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